

Neutrino Oscillations from Discrete Non-Abelian Family Symmetries

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Abstract

I discuss a SUSY-GUT model with a non-Abelian discrete family symmetry that explains the observed hierarchical pattern of quark and lepton masses. This $SO(10) \times \Delta(75)$ model predicts modified quadratic seesaw neutrino masses and mixing angles which are interesting for three reasons: i.) they offer a solution to the solar neutrino problem, ii.) the tau neutrino has the right mass for a cosmologically interesting hot dark matter candidate, and iii.) they suggest a positive result for the $\nu_\mu \rightarrow \nu_\tau$ oscillation searches by the CHORUS and NOMAD collaborations. However, the model shares some problems with many other predictive GUT models of quark and lepton masses. Well-known and once successful mass and angle relations, such as the $SU(5)$ relation $\lambda_b^{GUT} = \lambda_\tau^{GUT}$, are found to be in conflict with the current experimental status. Attempts to correct these relations seem to lead to rather contrived models.

1. Introduction

Fermion masses and mixing angles correspond to free parameters in the Standard Model (SM). It is widely believed that there should be a more general theory that predicts at least some of these parameters from first principles. Even though this problem has inspired many theorists to attempt a solution [1-4] we are still lacking a compelling theory. The obvious hierarchical pattern of the masses and mixing angles seems to suggest a possible explanation via a slightly broken symmetry [4].

It has been shown in a previous publication [5] that a non-Abelian family symmetry, with the three families transforming as an irreducible representation, can be used as a very powerful tool to constrain the Yukawa couplings of the SM, resulting in interesting fermion mass textures. In [5], it has been demonstrated that these symmetries naturally suppress flavor changing neutral currents in supersymmetric theories. Kaplan and Muryama have used non-Abelian symmetries to constrain “dangerous” proton decay operators [6].

The model presented as an example in [5] demonstrates nicely how interesting Yukawa matrix textures can be obtained from non-Abelian family symmetries. Unfortunately, due to the large number of unknown parameters entering the Yukawa coupling matrices, it does not give rise to any precise numerical predictions.

The model presented in this publication is based on the same approach - it is an $SO(10)$ SUSY-GUT with a non-Abelian family symmetry¹ - but is more ambitious: it predicts the light quark masses (m_s , m_d , m_u) as well as all neutrino mass ratios and lepton mixing angles. The reduction of parameters in this model relative to the one in [5] is due to a more efficient exploitation of the restrictive power of the $SO(10)$ gauge symmetry.

The $\Delta(75)$ family symmetry of the model determines the Yukawa matrix texture. At the GUT scale the three families are unified into the fundamental triplet representation of $\Delta(75)$. Below M_{GUT} the family symmetry is broken and the hierarchical pattern of Yukawa couplings is generated. The coupling strengths are determined by the charges of the various fields under the Z_3 and Z_5 subgroups of $\Delta(75)$. These charges allow only the top quark to have a renormalizable coupling to an $SU(2) \times U(1)$ breaking Higgs VEV; all other couplings arise at higher orders of $\Delta(75)$ breaking. The spontaneous family symmetry breaking is accomplished with a few non-trivial Higgs VEVs.

¹ In addition, a flavor blind $U(1)$ or R symmetry is required in order to forbid some unwanted couplings.

Once created by the family symmetry at the GUT scale, the hierarchical coupling patterns are protected by the non-renormalization property of supersymmetry.

The most interesting predictions of this model lie in the neutrino sector. The model, which has been constructed to fit the SM fermion masses and mixing angles, has a completely determined neutrino Dirac mass matrix Y_ν . All its components are related to entries in the quark and charged lepton matrices by the $SO(10)$ symmetry. Since the non-Abelian family symmetry constrains the Majorana mass matrices for the right handed neutrinos to be very simple (in this model, it is proportional to the unit matrix) one can unambiguously predict all the neutrino mass ratios and lepton mixing angles via the seesaw approximation [7]. The $SO(10)$ Clebsches modify the usual quadratic mass relations in an interesting way. One finds that

- i.) the predictions for $\sin^2(\Theta_{\nu_e\mu}) = 0.019 \pm 0.008$ and $m_{\nu_\mu} \sim \mathcal{O}(10^{-3})$ eV allow the small angle MSW solution to the solar neutrino problem,
 - ii.) the tau neutrino mass, $m_{\nu_\tau} \sim \text{few eV}$, allows the tau neutrino to play the role of the hot dark matter component in a mixed dark matter scenario [8], and
 - iii.) oscillations between muon and tau neutrinos may well be observable by the collaborations NOMAD and CHORUS at CERN [9]. I show the model's predictions compared to present and future experimental limits in a plot of the $\sin^2(\Theta_{\nu_\mu\tau}) - \Delta(m^2)$ plane.
- The Yukawa matrices of this model are similar to the well-known Georgi-Jarlskog (GJ) matrices [10] with a few small but important differences. The family symmetry leads to non-zero entries in the $\{2, 3\}$ and $\{3, 2\}$ components of the down quark and charged lepton Yukawas. These entries have the effect of lowering the prediction for $|V_{cb}|$ which in the GJ scheme is rather high. The other difference is a 20% correction to $\lambda_b^{GUT} = \lambda_\tau^{GUT}$ which stems from an operator that involves $SU(5)$ breaking VEVs. This contribution lowers the otherwise unacceptably high value obtained for $R = m_b/m_\tau$. A more detailed discussion of problematic mass and angle relations is left to the conclusions.

2. Fields and interactions

In this section, I present a specific supersymmetric $SO(10) \times \Delta(75)$ GUT which incorporates the features discussed in the introduction. The $\Delta(75)$ family symmetry constrains the allowed Yukawa couplings of the SM fermions, leading to a modified GJ texture.

Table 1 lists the superfields involved in the generation of quark and lepton masses. The three families of the SM are contained in the superfield \mathcal{F} . Then there are the fields

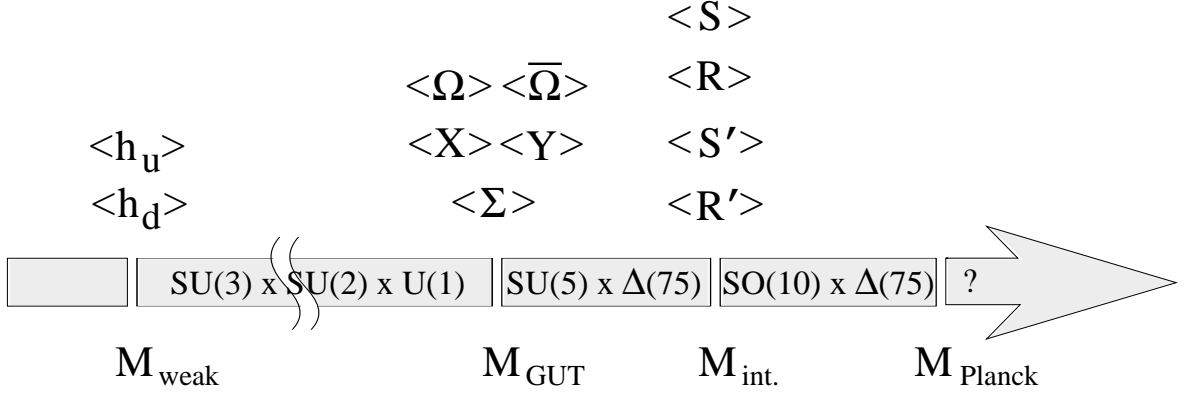


Fig. 1. *The various mass scales of the model and the symmetry breaking VEVs.*

ψ , $\bar{\psi}$ and χ , $\bar{\chi}$; they are superheavy and do not acquire VEVs. Various Higgs fields break the gauge and family symmetries in two steps. Figure 1 shows the mass scales at which the spontaneous symmetry breaking takes place. M_I is the scale where $SO(10) \times U(1)$ is broken to its subgroup $SU(5)$ by the VEVs of the fields S, S', R, R' , and the fields $\Psi, \bar{\Psi}$ and $\chi, \bar{\chi}$ obtain large masses. At M_{GUT} the gauge symmetry is further broken to the MSSM gauge group by the VEVs of Σ and Ω . At the same scale the flavor symmetry is broken by the VEVs of X, Y , and Σ . When the heavy “matter fields” $\psi, \bar{\psi}$ and $\chi, \bar{\chi}$ are integrated out of the theory and the flavor symmetry breaking Higgs fields acquire their VEVs they generate effective Yukawa couplings for the light fields. These couplings will be suppressed by varying powers of

$$\epsilon \simeq \frac{\langle X, Y, \Sigma \rangle}{(M_\psi, M_\chi)} \simeq \frac{M_{GUT}}{M_I}.$$

Finally, at the weak scale the Higgs doublets acquire their VEVs, thus giving the masses to the SM quarks and leptons.

Given the fields and representations in Table 1, the most general renormalizable superpotential consistent with the symmetries is

$$\begin{aligned} W_4 = & X\psi\mathcal{F} + \Sigma\bar{\psi}\mathcal{F} + H_d\bar{\chi}\Sigma + H'_d\psi\psi \\ & + \chi[\mathcal{F}\mathcal{F} + \mathcal{F}\psi] + H_u[\mathcal{F}\mathcal{F} + \mathcal{F}\psi], \end{aligned} \quad (2.1)$$

where I have suppressed all coupling constants, they are assumed to be $\mathcal{O}(1)$. Several remarks about this superpotential are in order:

Field	$SO(10)$	$\Delta(75)$	$Mass$	Field	$SO(10)$	$\Delta(75)$	$Mass$
\mathcal{F}	16	T_1	M_W	Σ	45	\bar{T}_4	M_{GUT}
$\Psi, \bar{\Psi}$	16, $\bar{16}$	\bar{T}_4, T_4	M_I	$\Omega, \bar{\Omega}$	16, $\bar{16}$	1	M_{GUT}
$\chi, \bar{\chi}$	10, $\bar{10}$	\bar{T}_2, T_2	M_I	X	1	\bar{T}_3	M_{GUT}
S	45	1	M_I	Y	1	\bar{T}_2	M_{GUT}
S'	1	1	M_I	H_u	10	\bar{T}_2	M_{GUT}^*
R	1	1	M_I	H_d	10	\bar{T}_3	M_{GUT}^*
R'	1	1	M_I	H'_d	10	\bar{T}_3	M_{GUT}^*

Table 1. *Fields and representations. Stars point out that the components of the H fields that correspond to the electroweak breaking Higgs h_u and h_d stay massless at M_{GUT} .*

1. I have omitted a $S\bar{\chi}H_u$ operator; it can be rotated away by a suitable redefinition of χ and H_u which carry identical quantum numbers.
2. The down type Higgs fields do not have renormalizable couplings to the SM fermions. This implies that the bottom Yukawa coupling is automatically suppressed compared to the top coupling, resulting in low $\tan\beta = \langle h_u/h_d \rangle$ and thus avoiding the problems associated with large $\tan\beta$ [11,12].
3. The down quark and lepton Yukawa couplings get contributions from two down type Higgs fields. Only a linear combination of the third flavor component of H_d and the first flavor component of H'_d remains light after the flavor symmetry breaking.

Since the GUT scale and the $SO(10)$ breaking scale are only a couple of orders of magnitude from the Planck scale, there are non-negligible contributions to the Yukawa couplings from operators of dimension greater than four. These operators arise from gravitational interactions and are suppressed by the appropriate powers of M_P :

$$W_{5+6} = \frac{1}{M_P} [\bar{\chi}Y\bar{\Omega}\bar{\Omega}] + \frac{1}{M_P^2} [H_u\mathcal{F}FYR + H_d\mathcal{F}FYR'] . \quad (2.2)$$

The first term is important for the masses of the right handed neutrinos, while the dimension six operators contribute to the first family Yukawa couplings.

In order to generate the MSSM with realistic masses for the quarks and leptons, it is necessary to make certain assumptions about the symmetry breaking pattern. I assume the following:

1. The fields S, S', R, R' acquire VEVs at the scale M_I which lies somewhere between M_{GUT} and M_P , giving large masses to the ψ and χ fields. The VEV of S also breaks $SO(10)$ down to $SU(5)$.
2. $SU(5)$ is further broken to $SU(3) \times SU(2) \times U(1)$ at M_{GUT} by VEV of Σ . Each flavor component of the field Σ develops an identical VEV. This also breaks the family symmetry $\Delta(75)$ to its subgroup Z_3 .
3. The family symmetry is broken completely by the fields X and Y . X develops a GUT scale VEV along its first component, thus leaving a Z_5 subgroup unbroken, while Y has identical VEVs in all three components. It is through the VEVs of X , Y , and Σ that mass mixing between the heavy fermions ψ , χ and the light fermions \mathcal{F} is induced.
4. The $SU(2) \times U(1)$ breaking VEVs are a little more complicated. I assume that only the $Y = -1/2$ weak doublet from $(H_u)_3$ and the weak $Y = +1/2$ doublet contained in a linear combination of $(H_d)_3$ and $(H'_d)_1$ remain lighter than M_{GUT} and participate in electroweak symmetry breaking. In the following, I denote the light Higgs doublets by h_u and h_d .

One can now determine the resulting effective Yukawa couplings just below M_{GUT} by calculating the diagrams with renormalizable couplings (Figure 2.a.) and with non-renormalizable couplings (Figure 2.b.).

3. Quark and charged lepton Yukawa couplings

One sees from the diagrams in Figure 2.a. that only the top quark field has a renormalizable coupling to the weak scale Higgs fields. All other quark and lepton Yukawa couplings involve flavor symmetry breaking and are suppressed. The second diagram contributes to the $\{2, 3\}$ and $\{3, 2\}$ entries of the up Yukawa matrix, and is suppressed by a factor of $\epsilon_x = \frac{\langle X \rangle}{M_\psi} \sim \frac{M_{GUT}}{M_I}$. The third diagram does not contribute to Y_u because of a vanishing $SO(10)$ Clebsch Gordan coefficient. The fourth through sixth diagrams are the corresponding diagrams for the down quark and charged lepton sector. They are suppressed by $\epsilon_\sigma = \frac{\langle \Sigma \rangle}{M_\chi} \sim \frac{M_{GUT}}{M_I}$ compared to the first three diagrams. The seventh diagram gives an additional contribution to the $\{3, 3\}$ entries of the down and charged lepton Yukawa couplings. This contribution is not $SU(5)$ symmetric and splits m_b and m_τ by an amount of $\mathcal{O}(20\%)$. The eighth diagram corresponds to a flavor off-diagonal effective D-term and requires wavefunction renormalization. However, wave function renormalization is negligible in this model.

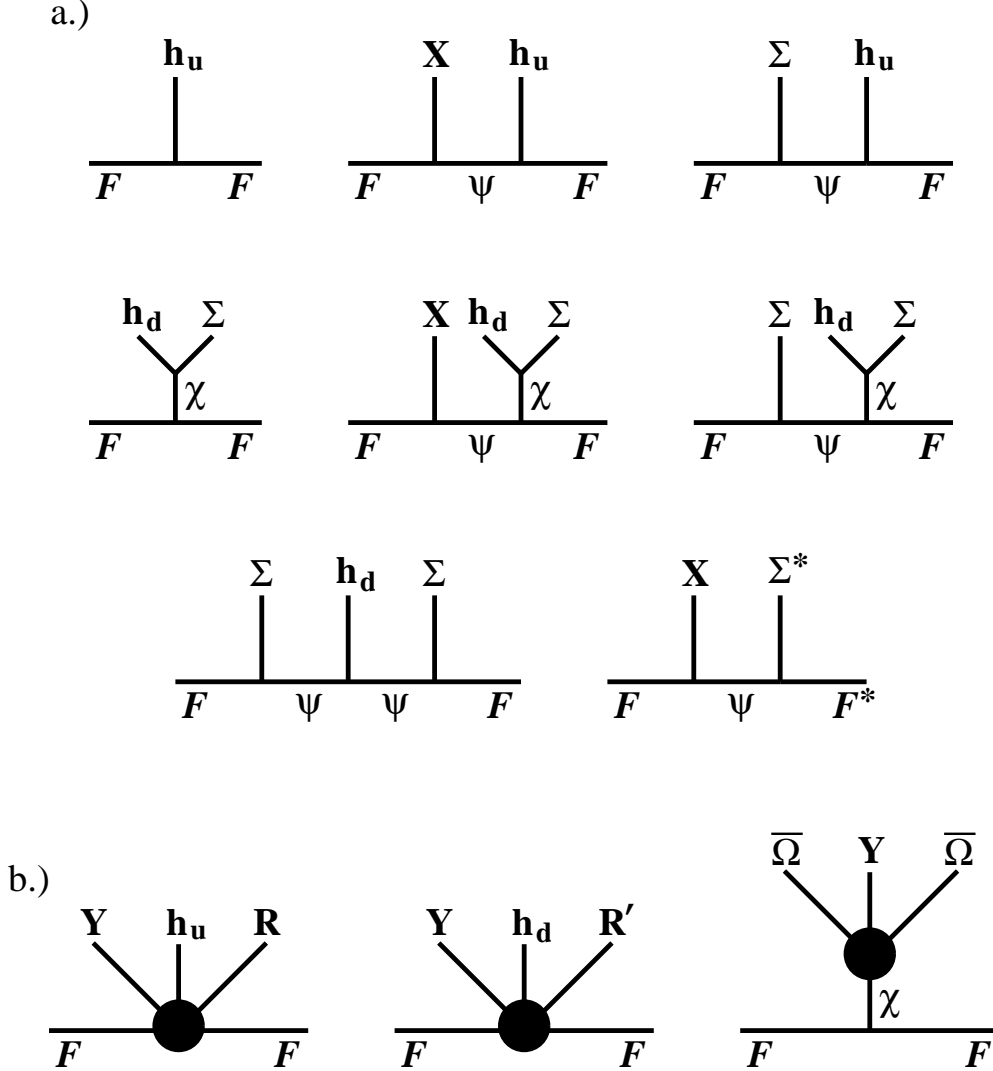


Fig. 2. Leading supergraph contributions to quark and lepton Yukawa couplings. Internal lines indicate ψ , $\bar{\psi}$, χ and $\bar{\chi}$ superfields. External lines are the light fermions \mathcal{F} and various Higgs fields. Black blobs represent non-renormalizable interactions suppressed by the appropriate powers of M_P . The last diagram in fig. 2.a. corresponds to an effective D -term.

The first and second diagrams in Figure 2.b. contribute to the $\{1,2\}$ and $\{2,1\}$ entries of the up and down Yukawa coupling matrices². They are suppressed by factors of $\delta = \frac{\langle R \rangle \langle Y \rangle}{M_P^2} \sim \frac{M_{GUT}}{M_P} \frac{M_I}{M_P}$.

² The third diagram does not contribute to SM fermion masses. It generates large masses for the right handed neutrinos.

Taking into account $SO(10)$ Clebsch Gordan coefficients, one obtains the following Yukawa coupling matrices

$$\begin{aligned}
Y_u &= \begin{pmatrix} 0 & \delta_u & 0 \\ \delta_u & 0 & \epsilon_x \\ 0 & \epsilon_x & A \end{pmatrix}, & Y_d &= \begin{pmatrix} 0 & \delta_d e^{i\phi_1} & 0 \\ \delta_d e^{-i\phi_1} & \frac{4}{3}\epsilon_\sigma \epsilon_{\sigma'} e^{i\phi_2} & \frac{1}{3}B\epsilon_x \epsilon_\sigma e^{i\phi_3} \\ 0 & B\epsilon_x \epsilon_\sigma e^{-i\phi_3} & |\epsilon_\sigma + \frac{1}{3}C\epsilon_{\sigma'}^2| \end{pmatrix}, \\
Y_l &= \begin{pmatrix} 0 & \delta_d e^{i\phi_1} & 0 \\ \delta_d e^{-i\phi_1} & -4\epsilon_\sigma \epsilon_{\sigma'} e^{i\phi_2} & -B\epsilon_x \epsilon_\sigma e^{i\phi_3} \\ 0 & -\frac{1}{3}B\epsilon_x \epsilon_\sigma e^{-i\phi_3} & |\epsilon_\sigma + 3C\epsilon_{\sigma'}^2| \end{pmatrix},
\end{aligned} \tag{3.1}$$

where Y_u^{ij} , Y_d^{ij} , and Y_l^{ij} are the coefficients of the effective operators $h_u Q_i U_j^c$, $h_d Q_i D_j^c$, and $h_d L_i E_j^c$ respectively, and where I have defined

$$\begin{aligned}
\epsilon_x &= \frac{\langle X_1 \rangle}{M_\psi}, & \epsilon_\sigma &= \frac{\langle \Sigma_1 \rangle}{M_\chi}, & \epsilon_{\sigma'} &= \frac{\langle \Sigma_1 \rangle}{M_\psi} = \frac{\langle \Sigma_2 \rangle}{M_\psi}, \\
\delta_d &= \frac{\langle Y_3 \rangle \langle R' \rangle}{M_P^2}, & \delta_u &= \frac{\langle Y_2 \rangle \langle R \rangle}{M_P^2}.
\end{aligned} \tag{3.2}$$

All the parameters denoted with capital letters are $\mathcal{O}(1)$. The parameters $\epsilon_x, \epsilon_\sigma$, and $\epsilon_{\sigma'}$ are expected to be $\mathcal{O}(10^{-1})$ or $\mathcal{O}(10^{-2})$ from their definitions. The δ 's are $\mathcal{O}(10^{-3})$ or $\mathcal{O}(10^{-4})$. Unphysical phases have been rotated away, and I have neglected the small difference in phase between the $\{3, 3\}$ entries of Y_d and Y_l . The remaining phases are expected to be $\mathcal{O}(1)$. I have given only the leading contributions to each entry, and ignore the negligible contributions from wavefunction renormalization to the $\{13\}$, $\{31\}$ and $\{11\}$ entries. One sees that there is a natural hierarchical structure to the masses, and that down-type quarks and leptons are automatically a factor of ϵ_σ more weakly coupled to the Higgs doublet than are up-type quarks, thus predicting small $\tan \beta$. Notice the $SO(10)$ Clebsch factors [3] appearing in the matrices:

1. Factors of $\frac{1}{3}$ in the $\{2, 3\}$ and $\{3, 2\}$ entries of Y_D and Y_L .
2. The third diagram in Fig. 2.a. does not contribute to Y_u because of a zero $SO(10)$ Clebsch factor while the corresponding diagram for the down sector gives a factor of 3 difference between the magnitudes of the $\{22\}$ entries in Y_d and Y_l . The factor of -3 plays the same role as the factor of -3 in the GJ mass matrices.
3. The corrections to the b and τ Yukawa couplings from the seventh diagram have different Clebsch factors, thus splitting λ_b and λ_τ at M_{GUT} .
4. The gravitationally induced interactions which contribute to the u , d , and e masses as well as to the Cabibbo angle do not contain any $SO(10)$ Clebsch factors.

m_e	m_μ	m_τ	m_c	m_b	m_t	$\frac{ V_{ub} }{ V_{cb} }$	$ V_{cb} $	$ V_{us} $
$5.11 \cdot 10^{-4}$	0.106	1.78	1.3 ± 0.3	4.3 ± 0.2	174 ± 16	0.08 ± 0.02	0.040 ± 0.005	0.221

Table 1. *Experimental input parameters are taken from [14].
Quark masses are displayed in units of GeV.*

4. Renormalization group evolution and numerical predictions

I now determine the parameters of the Yukawa coupling matrices in (3.1) by running them to the scale of the respective fermion masses, diagonalizing the mass matrices and matching onto the measured masses and mixing angles. For the evolution between M_{GUT} and m_t I use one-loop MSSM renormalization group equations. Between M_{GUT} and the scale of the Majorana masses of the right handed neutrinos M_N , which I take at 10^{12} GeV, one also needs to include the contributions to the running from the neutrino Yukawa coupling $\lambda_{\nu\tau}$ ³. Below the scale of the top quark mass I utilize three-loop QCD and one-loop QED scaling factors which I adopt from Babu and Mohapatra [13]: $(\eta_u, \eta_{d,s}, \eta_c, \eta_b, \eta_{e,\mu}, \eta_\tau) = (2.49, .48, 2.17, 1.55, 1.02, 1.02)$, where $\eta_f = m_f(m_f)/m_f(m_t)$ for $f = c, b, \tau$, and $\eta_f = m_f(1 \text{ GeV})/m_f(m_t)$ for the light fermions. The experimental input parameters are listed in Table 2.

The renormalization procedure for the Yukawa couplings is well-known and has been performed in a number of publications [2-4,15]. I only mention some important features. Most models based on $SU(5)$ or $SO(10)$ lead to the boundary condition $\lambda_b^{GUT} = \lambda_\tau^{GUT}$, and one determines λ_t^{GUT} through its important contribution to the running of $R(\mu) = m_b(\mu)/m_\tau(\mu)$. Using a representative value for $\alpha_s(M_Z) = 0.12$, one finds a very high value for $\lambda_t^{GUT} \sim 3$. While this possibility cannot be ruled out, it does constitute a serious problem to any predictive GUT extension because the large top Yukawa coupling becomes infinite closely above the GUT scale. For example, $\lambda_t^{GUT} = 3$ leads to a Landau pole at $2 M_{GUT}$ in both $SU(5)$ and $SO(10)$. As a result, one loses predictivity completely because now one expects higher dimension operators “suppressed” by factors of $M_{GUT}/2M_{GUT}$ to play an important role. Demanding perturbativity up to M_{Planck} requires $\lambda_t^{GUT} \leq 1.3$ in this model, and one is forced to give up and correct the $SU(5)$ relation $\lambda_b^{GUT} = \lambda_\tau^{GUT}$.

³ For simplicity, I assume $M_{SUSY} = m_t$. I also ignore contributions from λ_b and λ_τ to the evolution equations, they are negligible in a small $\tan\beta$ scenario. I have checked that using two loop renormalization group equations does not change the results significantly.

When including the partially cancelling contributions to the running from both $\lambda_{\nu_\tau}^{GUT} = 1.3$ and $\lambda_t^{GUT} = 1.3$ I find $R(M_{GUT}) = 0.85$ from the experimental input ⁴. In the following, I fix $\lambda_t^{GUT} = 1.3$ in order to maximize its contribution to the running of R . The predictions of the model are not very sensitive to variations of λ_t as long as $\lambda_t^{GUT} \leq 1.3$.

I now extract $\tan \beta, \epsilon_x, \epsilon_\sigma, \epsilon_{\sigma'}$ from m_t, m_τ, m_μ, m_c , respectively. The parameter B can be determined from $|V_{cb}^{GUT}| = \frac{\epsilon_x}{A} |1 - \frac{AB}{3} e^{i\phi_3}|$. This constrains $0.62 \leq |B| \leq 4.2$, but I will limit $B \leq 2$ because i.) larger values are disfavored by the experimental limits on $\nu_\mu \rightarrow \nu_\tau$ oscillations as I will show in the following section, and ii.) a value of $B \sim 1$ is favored from a theoretical viewpoint since B is defined as a combination of $\mathcal{O}(1)$ coupling constants. From the equations for $|V_{cb}|$ and $|V_{us}|$ one can also determine the phases $|\phi_3|$ and $|\phi_1 - \phi_2|$. However, this does not suffice to predict CP violation because of the unconstrained phase $|\phi_1 + \phi_2|$. Finally, δ_u and δ_d are determined from $|V_{ub}|/|V_{cb}|$ and m_e . Numerically these parameters are

$$\begin{aligned} A &= 1.3, \quad 0.62 \leq B \leq 2, \quad |1 + 3C \frac{\epsilon_{\sigma'}^2}{\epsilon_\sigma}| = 1.20, \\ \epsilon_x &= 5.6 \pm 0.8 \cdot 10^{-2}, \quad \epsilon_\sigma = 1.30 \cdot 10^{-2}, \quad \epsilon_{\sigma'} = 1.67 \cdot 10^{-2}, \\ \delta_u &= 1.9 \cdot 10^{-4}, \quad \delta_d = 0.51 \cdot 10^{-4}, \quad \tan \beta = 1.94. \end{aligned} \tag{4.1}$$

The parameters A, B , and C are of $\mathcal{O}(1)$, as expected. This means that $\Delta(75)$ is “working properly”, that is, no unnaturally large or small couplings in the superpotential are necessary. The hierarchy of Yukawa couplings is entirely explained as powers of $\Delta(75)$ symmetry breaking VEVs over intermediate particle masses or the Planck scale.

The model leads to three predictions in the quark and charged lepton sector:

$$m_s = \frac{|1 - 2\xi|}{3\eta} \frac{\eta_s}{\eta_\mu} m_\mu 188 \pm 3\% B^2 \text{ MeV} , \tag{4.2}$$

$$\frac{m_d}{m_s} = 9 |1 + 2\xi|^2 \frac{m_e}{m_\mu} = \frac{1}{22.9} \pm 6\% B^2 , \tag{4.3}$$

$$m_u = \left(\frac{V_{ub}}{V_{cb}} \right)^2 \frac{\eta_u}{\eta_c} m_c = 9.5 \pm 5.2 \text{ MeV} . \tag{4.4}$$

Here $\eta = 0.45$ is an evolution factor that accounts for the running of $\frac{m_s}{m_\mu}$ from the GUT scale down to the weak scale. η_u, η_c, η_μ , and η_s have been given before, and $\xi = \frac{B^2 \epsilon_x^2}{12 \epsilon_{\sigma'}} e^{-i\gamma}$

⁴ The connection between the mass scale of the right handed neutrinos and the m_b/m_τ ratio has been pointed out in [16].

is small. It varies between $6.0 \cdot 10^{-3}$ and $6.2 \cdot 10^{-2}$ as B is varied from 0.62 to 2. The predictions should be compared to the estimates from chiral perturbation theory [14]. The value for m_d/m_s agrees very well, while the value for $m_u/m_d = 1.16 \pm 55\%$ is quite large and is only consistent because of the large uncertainties in the prediction which stem from the experimental error bars of the input value for $|\frac{V_{ub}}{V_{cb}}|$.

5. Neutrino masses

The field \mathcal{F} that contains all the SM fermions also contains an $SU(3) \times SU(2) \times U(1)$ singlet field that plays the role of the Dirac partner N^c of the left handed neutrino in the lepton doublet. The $SO(10)$ symmetry relates the neutrino Yukawa coupling matrix Y_ν^{ij} to the up quark Yukawa matrix by known Clebsch Gordan coefficients

$$Y_\nu = \begin{pmatrix} 0 & \delta_u & 0 \\ \delta_u & \frac{8}{5} \frac{\epsilon_{x'}}{B} e^{i(\phi_2 + \phi_3)} & \frac{1}{5} \epsilon_x \\ 0 & \frac{1}{3} \epsilon_x & A \end{pmatrix}. \quad (5.1)$$

All the components are given in terms of parameters already determined from the quark and charged lepton sector. Note that unlike the corresponding up Yukawa matrix, the $\{2, 2\}$ component of Y_ν^{ij} does not have a vanishing Clebsch factor. This leads to an interesting modification of the usual quadratic seesaw mechanism [7,17]. The effective Majorana mass of the neutrinos as we would measure it in a successful neutrino oscillation experiment is then given by

$$M_\nu = - \left[\frac{v \sin \beta}{2} \right] Y_\nu M_N^{-1} Y_\nu^T \quad (5.2)$$

where M_N^{-1} is the inverse of the Majorana mass matrix of the heavy right handed neutrinos. In general M_N is completely arbitrary. But in a model such as this one where the fermions transform as irreducible triplet representations of the family symmetry, the form of M_N is very restricted. $\Delta(75)$ predicts it to be either proportional to the unit matrix or else completely off-diagonal with all identical entries. In either case the resulting M_N^{-1} is non-hierarchical and completely determined except for the overall multiplicative mass scale. In this model, the third diagram in Figure 2.b. leads to M_N proportional to the unit matrix

with an overall factor $\langle Y \rangle \langle \Omega \rangle^2 / M_P M_I \sim M_{GUT}^3 / M_P M_I \sim 10^{12}$ GeV. Diagonalizing, I find the following predictions for the neutrino mass ratios and lepton mixing angles:

$$\begin{aligned} \frac{m_{\nu_\mu}}{m_{\nu_\tau}} &= \left(\frac{8}{5} \frac{\epsilon_{\sigma'} \eta_\nu}{AB} \right)^2 \simeq 6.5 \cdot 10^{-4} B^{-2} , \\ \frac{m_{\nu_e}}{m_{\nu_\mu}} &= \frac{\delta_u^4 \eta_\nu^4}{A^4} \left(\frac{m_{\nu_\tau}}{m_{\nu_\mu}} \right)^2 \simeq 2.6 \cdot 10^{-9} B^4 , \end{aligned} \quad (5.3)$$

$$\begin{aligned} |\Theta_{\nu_e \mu}| &= \sqrt{\frac{m_e}{m_\mu}} \simeq 0.069 \pm 0.007 B , \\ |\Theta_{\nu_\mu \tau}| &= B \epsilon_x \left| \eta_N + \frac{\eta_\nu e^{-i\beta}}{5AB} \right| \simeq B \epsilon_x \left(\eta_N + \frac{\eta_\nu}{5AB} \right) \simeq 0.059 B + 0.011 , \\ \left| \frac{\Theta_{\nu_e \tau}}{\Theta_{\nu_\mu \tau}} \right| &= \left(\frac{m_{\nu_e}}{m_{\nu_\mu}} \right)^{1/4} \simeq 0.007 B , \end{aligned} \quad (5.4)$$

with the evolution factors $\eta_\nu = 1.24$ and $\eta_N = 1.06$. The overall mass scale is approximately given by $m_{\nu_\tau} \sim \text{few eV}$ and therefore $m_{\nu_\mu} \sim 10^{-3} \text{eV}$. While the prediction for m_{ν_τ} allows the tau neutrino to play the role of a hot dark matter candidate in a mixed dark matter scenario, the model also offers a solution to the solar neutrino problem via $\nu_e \leftrightarrow \nu_\mu$ oscillations.

This suggests that we use the experimental value for Δm^2 from the MSW solution to the solar neutrino problem, $m_{\nu_\mu} = 1.8 \cdot 10^{-3} - 3.5 \cdot 10^{-3}$ eV, as input in order to fix the overall massscale [18]. The resulting predictions for $\nu_\mu \rightarrow \nu_\tau$ oscillations are plotted in Figure 3. together with the present limits from accelerator oscillation experiments and the expected sensitivities for the new generation of experiments, NOMAD and CHORUS at CERN. One finds that large values of $B \geq 1.2$ are already ruled out and the exciting prospect that NOMAD and CHORUS may soon see the first direct evidence for neutrino oscillations.

For completeness, I also mention that the model's predictions for $\nu_e \leftrightarrow \nu_\tau$ oscillations are far from current experimental limits due to the small $\Theta_{\nu_e \tau}$ mixing angle, and that they are consistent with more stringent limits derived from heavy element nucleosynthesis in supernovae [19].

6. Conclusions

In this letter I have presented a new SUSY-GUT model which predicts fermion masses and mixing angles. The non-Abelian family symmetry group of the model explains the

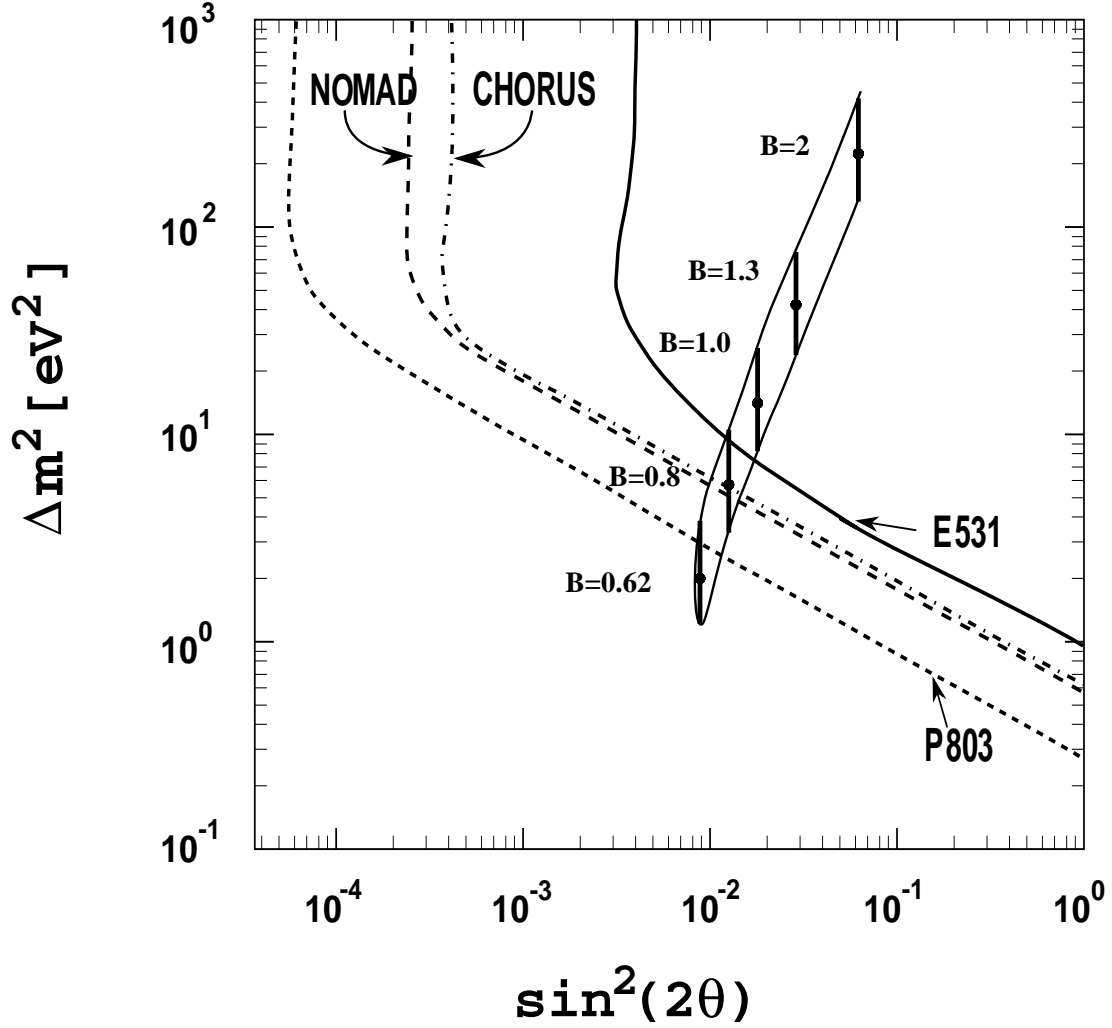


Fig. 3. Predictions for $m_{\nu\tau}^2$ and $\sin^2(2\Theta_{\nu\mu\tau})$ compared to limits from past and future $\nu_\mu - \nu_\tau$ oscillation experiments [9]. The predictions for $\Theta_{\nu\mu\tau}$ and $m_{\nu\tau}$ increase with increasing B .

observed hierarchical and diverse spectrum of masses and angles in terms of the hierarchy between the mass scales M_{GUT} , M_{Planck} , and an intermediate scale M_I where $SO(10)$ is broken down to $SU(5)$. The numerical predictions arise because the $SO(10)$ symmetry relates entries from different Yukawa matrices via Clebsch Gordan coefficients.

Particularly interesting is the $SO(10)$ Clebsch structure in the $\{2, 2\}$, $\{2, 3\}$, and $\{3, 2\}$ components of the Yukawa matrices. These Clebsches, while ensuring consistency with measured SM masses and angles, also show up in the neutrino sector and lead to predictions which are more successful than the naive quadratic seesaw relations.

However, while the model meets the goal of generating a Yukawa texture that is predictive and in accord with all experimental data, it does so only at the cost of a rather complicated symmetry breaking sector. This is due to problems that seem to be generic. Many of the mass and angle relations that have been derived from various “standard textures” such as the Fritzsch texture or the GJ texture do not work at the level of precision at which we know SM parameters today. I list three such problematic relations:

1. The most “annoying” problem in the context of an $SU(5)$ or $SO(10)$ theory is that $\lambda_b^{GUT} = \lambda_\tau^{GUT}$ unification does not lead to a believable prediction for m_b/m_τ . The problem is the following. The renormalization group equation for $R(\mu) = m_b(\mu)/m_\tau(\mu)$ depends crucially on a large top Yukawa coupling⁵. A prediction consistent with experiment requires $\lambda_t^{GUT} \sim 3$. However, such a large Yukawa coupling leads to a Yukawa Landau pole closely above the GUT scale ($2M_{GUT}$). This opens a Pandora’s box of nearly unsuppressed higher dimensional operators which are expected to arise from the non-perturbative physics, and predictivity is lost completely. Alternatively, one could limit $\lambda_t^{GUT} \leq 1.3$ and avoid a Landau pole below M_{Planck} at the cost of giving up $\lambda_b^{GUT} = \lambda_\tau^{GUT}$. However, the necessary $\mathcal{O}(15\%)$ corrections to R introduce a new parameter and loss of predictivity. Also, this fix renders the model more complicated because it is not easy to move the “cornerstone” of an $SU(5)$ Yukawa theory which really sits at $R(M_{GUT}) = 1$ ⁶. In an $SO(10)$ theory the situation is further worsened by a cancellation of the top contribution to the β function for R by an identical contribution from λ_{ν_τ} which enters with opposite sign. The lower the scale of the right handed neutrino masses, the larger (worse) the prediction for R . For a review see [16].
2. A problem for models based on the GJ texture is the high value predicted for $V_{cb} \simeq 0.050$ ⁷. In the context of family symmetries this relation finds an easy and rather natural fix via additional entries in the down quark matrix.
3. The last problem I want to mention is the relation $\sqrt{\frac{m_u \eta_c}{m_c \eta_u}} = \left| \frac{V_{ub}}{V_{cb}} \right|$. It arises from all textures with zeros in the $\{1,1\}$, $\{1,3\}$, and $\{3,1\}$ components of the Yukawa

⁵ I am implicitly assuming small $\tan \beta$ in ignoring contributions from λ_b and λ_τ . For large $\tan \beta$ the prediction of m_b/m_τ has recently been shown to be problematic as well [12].

⁶ λ_b^{GUT} and λ_τ^{GUT} can only be split by the VEV of a $\bar{45}$ of $SU(5)$. The $\bar{45}$ could either be an additional down type Higgs field (dangerously large contribution to the gauge β function), or it could be a more complicated product of Higgs fields.

⁷ This prediction is especially high in the case of Yukawa trinification (large $\tan \beta$) [1,13].

matrices. This relation, while not being excluded, is disfavored because it predicts a rather high value for $m_u \simeq 9.5 \pm 5.2$ MeV. If combined with the GJ prediction for $m_d \sim 8$ MeV this results in $m_u/m_d = 1.2 \pm 0.6$.

It is encouraging to see that non-Abelian family symmetries lead to interesting textures with predictions that are very similar to the real world. However, it is frustrating to see that as the SM parameters are measured more and more accurately, the models that are in agreement with all data become increasingly complicated and less appealing. A successful predictive SUSY $SO(10)$ or $SU(5)$ GUT model will have to include a solution to the m_b/m_τ problem and probably new Yukawa matrix textures.

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